Triple Unification of Inflation, Dark matter and Dark energy in Chaotic Braneworld Inflation

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Abstract

In this paper, we show that in the framework of chaotic braneworld inflation, after preheating, the remaining oscillating inflaton field can play the role of dark matter with the observed level. Augmented by a non-zero effective cosmological constant Λ_4 on the brane, triple unification of inflation, dark matter and dark energy by a single field is realized. Our model perhaps is the simplest one in the market of theories to achieve triple unification.

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1   Introduction

The almost scale invariant spectrum of Gaussian adiabatic density perturbations observed from Cosmic Microwave Background (CMB) by WMAP satellite support the idea of inflation as the standard paradigm for the early universe. On the other hand, the "dark side of the universe" (dark energy and dark matter) currently dominate the energy budget of our universe. Usually we use different models to explain inflation, dark matter and dark energy, however in the interesting papers [1, 2], it was suggested that inflation, dark matter and dark energy can be unified by a single scalar field ϕ with the potential of the form

$$V = V_0 + \frac{1}{2}m^2\phi^2, \tag{1}$$

where V_0 is the dark energy we see today which may be chosen from the string landscape, ϕ plays the role of an inflaton field during inflation. This potential was also considered in [3] to unify inflation and dark energy. After inflation, presumably there is a subsequent preheating [4] through a coupling $g\phi^2\chi^2/2$ to some other scalar field χ, which makes the inflaton decays "incompletely". This reduces the field value to $\phi = m/g$ and the remaining inflaton hopefully will play the role of dark matter. Unfortunately, this simple and elegant scenario does not work because m, which is fixed by CMB temperature fluctuation, is too large so that unless $g = 10^7$ [2], the reduced field value will be too large to make a successful dark matter. Therefore extra mechanism to reduce the field value is needed. For example, in [1], a subsequent thermal inflation is assumed to further

reduce the field value and in [5, 6], the authors suggest that plasma mass effects [7] could provide the mechanism for an incomplete reheating after inflation.

As shown in [8], the inflaton mass m in chaotic inflation can be suppressed to be much lower if we consider the effect of modification to the Friedmann equation in brane cosmology (an application of this result can be found in [9]). Therefore it is worth to investigate whether m can be small enough and the inflaton field value can be reduced to the right value in order to play the role of dark matter with reasonable value of g. In this paper, we show the answer is positive. *The point is conventional chaotic inflation plus preheating does NOT work for a successful triple unification scenario, but chaotic braneworld inflation plus preheating simply works.*

The paper is organized as follows. In section 2 We briefly review the basic results of chaotic inflation on the brane which we call chaotic braneworld inflationin. In section 3, we present our main result and show why it can make a simple triple unification. Our conclusions are summarized in Section 4.

2   Chaotic Braneworld Inflation

In the braneworld inflation scenario where Einstein's equations hold in the five-dimensional bulk and the matter fields are confined to the 3-brane, four-dimensional Einstein equations and Friedmann equation are modified [10, 11, 12]. It is well known that the Friedmann equation in this context is given by [8]

$$H^2 = \frac{8\pi}{3M_P^2}\rho\left(1+\frac{\rho}{2\lambda}\right),$$
(2)

where $M_P = 1.22 \times 10^{19}$ GeV is the 4D Planck mass and λ is the brane tension. Eq. (2) reduces to the usual Friedmann equation at $\rho \ll \lambda$. The Planck mass in four and five dimensions is related by λ via

$$M_P = \sqrt{\frac{3}{4\pi}}\frac{M_5^3}{\sqrt{\lambda}}.$$
(3)

A lower bound of $M_5 \gtrsim 10^8$ GeV $= 8.2 \times 10^{-12}M_P$ is given by requiring the theory to reduce to Newtonian gravity on scales larger than 1 mm [8].

The curvature perturbation is given by

$$\zeta = \frac{H\delta\phi}{\dot\phi}.$$
(4)

If the mass of the field ϕ is much smaller than Hubble parameter, the field fluctuations at horizon exit are given by $\langle\delta\phi^2\rangle \simeq (H/2\pi)^2$. The spectrum of scalar perturbation at horizon exit is

$$A_s^2 \equiv \frac{4}{25}\langle\zeta^2\rangle = \frac{512\pi}{75M_P^6}\frac{V^3}{V'^2}\left(1+\frac{V}{2\lambda}\right)^3.$$
(5)

In the slow-roll approximation, the total number of e-folds during inflation is given by

$$N \equiv \int_{t_i}^{t_f} H dt \simeq -\frac{8\pi}{M_P^2}\int_{\phi_i}^{\phi_f}\frac{V}{V'}\left(1+\frac{V}{2\lambda}\right)d\phi,$$
(6)

where i and f are the values at the beginning and end of inflation, respectively. The value ϕ_f is obtained from the condition $\max\{\epsilon,|\eta|\} = 1$, where ϵ and η are the slow-roll parameters, given by

$$\epsilon \equiv \frac{M_P^2}{16\pi}\left(\frac{V'}{V}\right)^2\frac{1+V/\lambda}{(1+V/2\lambda)^2},$$
(7)

$$\eta \equiv \frac{M_P^2}{8\pi} \frac{V''}{V} \frac{1}{1 + V/2\lambda}. \tag{8}$$

Using Eq. (1) and (5) and imposing CMB normalization $A_s(\phi(N = 60)) \approx 2 \times 10^{-5}$ [13], we obtain

$$m \approx 4.5 \times 10^{-5} M_5. \tag{9}$$

This result is valid for $V/\lambda \gg 1$ which gives an upper bound on M_5, namely, $M_5 \ll 10^{17}$ GeV $= 8.2 \times 10^{-3} M_P$. The V_0 in Eq. (2) is the effective cosmological constant Λ_4 on the brane which is given by

$$V_0 = \Lambda_4 = \frac{4\pi}{M_5^3} \left(\Lambda + \frac{4\pi}{3M_5} \lambda^2 \right). \tag{10}$$

3   Triple Unification

After inflation, the scalar field start to oscillate at t_*, for $t > t_*$, we have [1]

$$\rho_\phi = \frac{1}{2} m^2 \phi_*^2 \left(\frac{a_*}{a} \right)^3, \tag{11}$$

and the radiation evolves as

$$\rho_R = \rho_R^* \left(\frac{a_*}{a} \right)^4, \tag{12}$$

where $\rho_R^* = 3M_P^2 m^2/8\pi$. We assume here an adiabatic expansion where the entropy $S = gT^3a^3$ is constant during the evolution and g is the entropic degrees of freedom. Then the number density of radiation is

$$n_{\gamma,0} = n_\gamma^* \frac{g_*}{g_0} \left(\frac{a_*}{a_0} \right)^3, \tag{13}$$

where the zero subscript indicates current values. By using $\rho_R^* = \pi^2 g_* T^4/30$ and $n_\gamma = 2\zeta(3)T^3/\pi^2$, we can obtain the current dark matter per photon ratio as [1]

$$\xi_{dm,0} \equiv \frac{\rho_{\phi,0}}{n_{\gamma,0}} \simeq 4 \frac{g_0}{g_*^{1/4}} \left(\frac{m}{M_P} \right)^{1/2} \frac{\phi_*^2}{M_P^2} M_P. \tag{14}$$

We can calculate $n_{\gamma,0}$ from current Cosmic Microwave Background (CMB) black body temperature and $\rho_{\phi,0}$ from the cold dark matter energy density Ω_c, hence a value $\xi_{dm,0} = 2.2 \times 10^{-28} M_P$ can be obtained from observation , which for typical values $g_* \sim 100$ and $g_0 = 3.9$ gives [13]

$$\left(\frac{m}{M_P} \right)^{1/2} \frac{\phi_*^2}{M_P^2} \simeq 4 \times 10^{-29}. \tag{15}$$

After preheating, $\phi = m/g$. We demand

$$\phi = \phi_*. \tag{16}$$

If the above condition is satisfied, naturally after preheating the original inflaton field will play the role of dark matter model as Eq. (15) is satisfied, *this is also necessary in order to permit a long radiation-dominated epoch and where the "conventional chaotic inflation plus preheating"*

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[1]In brane inflation scenario, the continuity equation is not changed [14].

scenario fails, because a too large ϕ is obtained after preheating and a further reduction of the field amplitude is required. However, in our case by using Eq. (9) and (15), we obtain

$$g = 5.83 \times 10^8 (M_5/M_P)^{5/4}. \tag{17}$$

The result is clearly shown in Fig. (1). As we can see, there is a regime $10^{-11} \lesssim M_5/M_P \lesssim 10^{-7}$ where we have $10^{-5} \lesssim g \lesssim 1$. This is a perfectly reasonable range both for conventional preheating or brane preheating [15][2]. Therefore in our case, no further reduction of ϕ after preheating is needed. After preheating, our universe enters the era of "hot big bang" via reheating and becomes radiation dominated and the remaining oscillating inflaton field becomes dark matter until now.
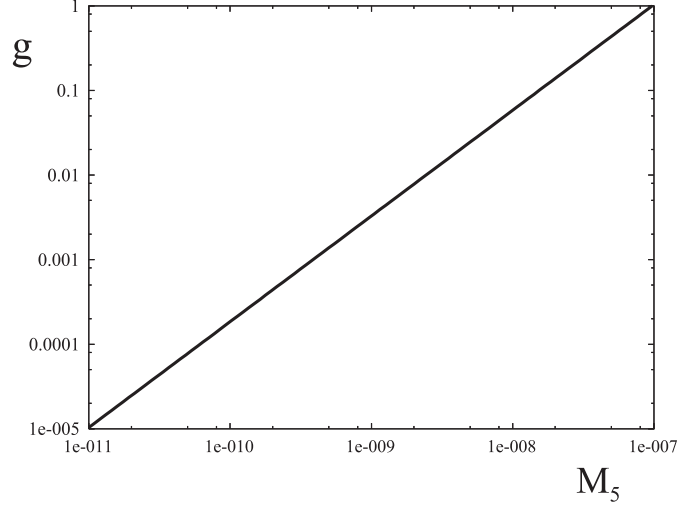

Figure 1: g versus M_5 with unit M_P

4   Conclusion

We have shown that triple unification of inflation, dark matter and dark energy can simply be achieved in the framework of chaotic braneworld inflation plus preheating. The required 5D Planck mass is in the range $10^{-11} \lesssim M_5/M_P \lesssim 10^{-7}$ corresponds to the coupling for reheating $10^{-5} \lesssim g \lesssim 1$. This model is very simple (perhaps the simplest one among all theories of triple unification) but it elegantly realized the idea of triple unification because no further reduction mechanism (for example, thermal inflation as in [1]) for the inflaton field ϕ is needed.

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[2]Our result do not depend on which kind of preheating we use.

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